Doppler shift estimation for 10 GHz aircraft enhancement

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Introduction

Doppler shift occurs during all Aircraft Enhanced (AE) radio contacts but the shifts in frequency go unnoticed on SSB on VHF and UHF. At 10 GHz the shifts are sufficient to be very noticeable on SSB and even more so if communication using JT65 is attempted. The question arises what magnitude of shift is to be expected and how do the shifts vary. Observation of a beacon propagated via numerous aircraft flights for which flight data is available can answer this question but a more convenient approach is to calculate the Doppler shift for a number of scenarios. This is the topic of this article.

The Calculations

The Doppler frequency shift D_f in Hz is given by the well-known equation:

$D_f = v/c * F_r * \cos(\Theta)$ where

v = aircraft speed, m/s

c = speed of radio waves, m/s

F_r = frequency of radio transmission, Hz

Θ = angle between the aircraft speed vector and a line to the transmitter, radian

The cos(Θ) term is necessary to account for the aircraft crossing the Great Circle Line (GCL) between stations at an angle.

The following initial assumptions were made:

Aircraft altitude: 36,000 ft

Aircraft speed: 500 knots

Distance between 2 stations: 600 km

Aircraft crosses GCL at the midpoint between stations

Antennas are aimed along the GCL and not moved

Antenna effective beam widths are ± 3°

The Earth is flat

These assumptions are sufficient as a starting point and are based on a typical situation for 10 GHz AE dx contacts. Assuming a flat earth does not introduce significant errors when the aircraft crosses the GCL at or near 90 degrees because the narrow beamwidth of the antennas limits the distances from the GCL over which AE is possible. Separate calculations for flights that track near the GCL have been made taking the earth's curvature into account because the distance covered by the aircraft, when AE is possible, is longer and the curvature effect is measurable.

Assuming a flat earth allows the use of two dimensional geometry for a variety of crossing angles, thus simplifying the analysis.

Figure 1 shows the geometry used for the calculations which were made using EXCEL© spreadsheets.

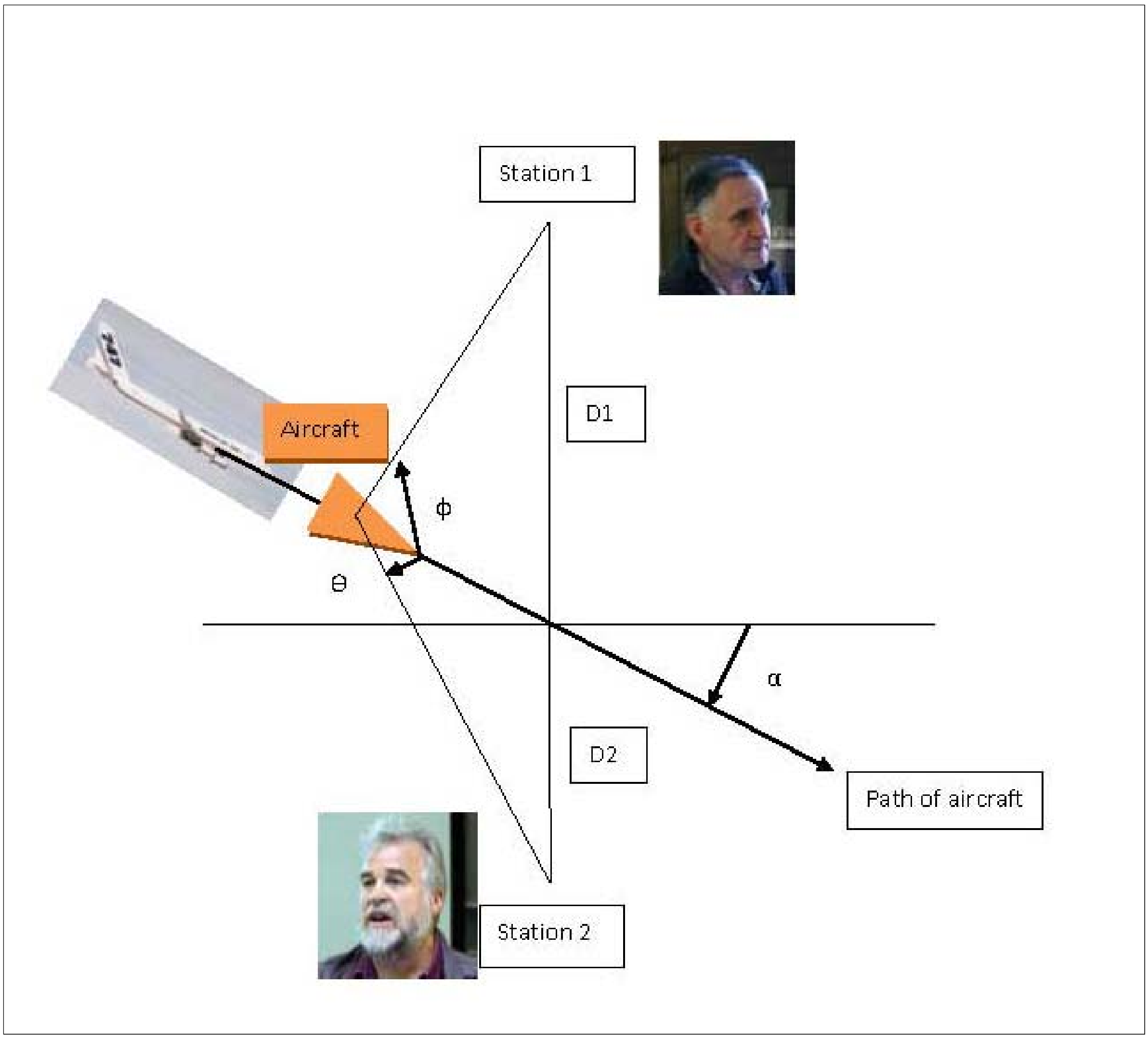

Figure 1. General geometry for an aircraft flying at constant altitude and constant speed crossing the perpendicular to the GCL between stations at an angle, α.
This arrangement was based on the use of East/West or West/East flights by stations north and south of the flight path. The angle α is zero when the aircraft flight path is perpendicular to the GCL.

D1 is the distance from station 1 to the flight crossing point on the GCL

D2 is the distance from station 2 to the flight crossing point on the GCL

ϕ is the angle between the flight path and the direct line to station 1 from the aircraft

Θ is the angle between the flight path and the direct line to station 2 from the aircraft

The calculations involved calculating ϕ and Θ for $\alpha = 0$ and the initial condition of the aircraft just illuminated by the edge of the station 1 antenna pattern. That is for Figure 1 and station 1 the angle from the GCL to the aircraft was + 3 degrees. This was taken to be the acquisition of signal position. The Doppler shift for the signal as received by station 2 involves adding the Doppler shift of the signal arriving at the aircraft to the Doppler shift of the signal departing the aircraft. The parameter α which is the deviation from a 90 degree crossing was chosen as it is the deviation from the worst case condition.

The Doppler shift is thereby reduced to a matter of geometry and change of range. For $\alpha = 0$ as the aircraft approaches the GCL the range to station 1 is reducing so the Doppler shift is positive. The range to station 2 is also decreasing so the associated Doppler shift is also increasing and the result for station 2 for is twice the single Doppler shift. A series of calculations were made with the aircraft advanced a little further along the flight path until the assumed loss of signal position was reached. The results were then put into a graph for easier assimilation.

At signal acquisition the Doppler shift is nearly +1 kHz shift for the assumed conditions. The Doppler shift decreases as the GCL crossing is approached. It is zero at the crossing of the GCL and goes negative to about 1 kHz at loss of acquisition of the signal which occurs about 120 seconds after initial acquisition of the signal. The rate of change of the Doppler was found by a regression line fit and is about 15.25 Hz per second. See Figure 2.

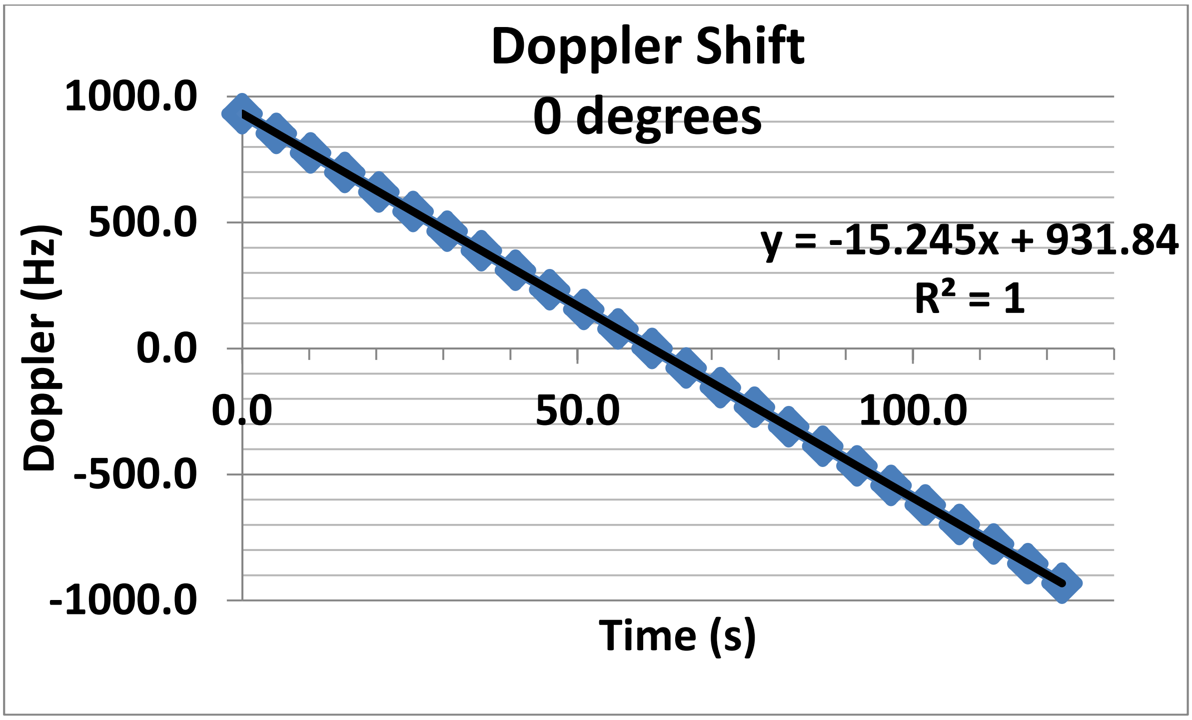

Figure 2. The calculated Doppler shift for α = 0. The aircraft is at 36,000 ft crossing the CGL at right angles at a point 300 km from station 1 And 300 km from station 2. The aircraft speed is 500 knots. Transmission frequency is 10 GHz. Maximum Doppler shift is about 930 Hz. Rate of change of Doppler is – 15.25 Hz/s. If α is negative the signs of the Doppler shifts reverse.

The values in Figure 2 are representative of all AE producing flights with a flight path that crosses the GCL at or near a right angle at or near the midpoint between stations. To investigate the effect of change in crossing angle the angle of crossing was changed to 45 degrees and the series of calculations redone. The results are shown in Figure 3.

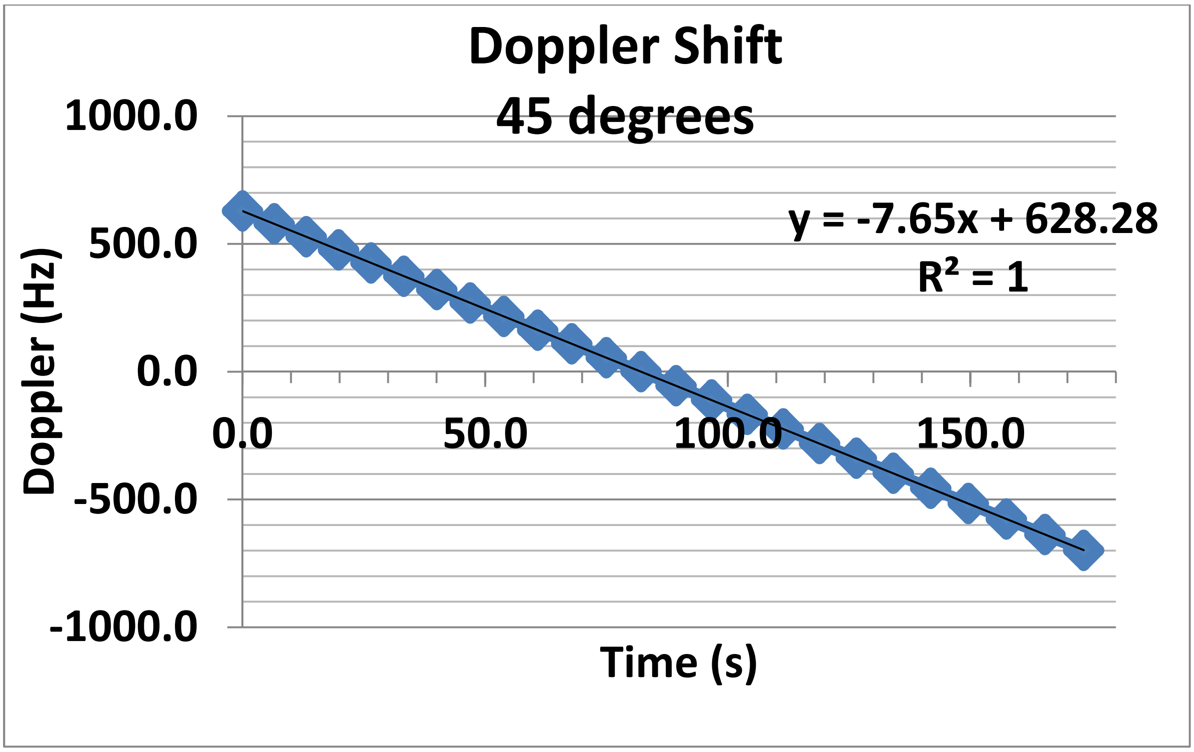

Figure 3. The calculated Doppler shift for α = 45° The aircraft is at 36,000 ft crossing the CGL at right angles at a point 300 km from station 1 And 300 km from station 2. The aircraft speed is 500 knots. Transmission frequency is 10 GHz. Maximum Doppler shift is about 630 Hz. Rate of change of Doppler is – 7.65 Hz/s

This suggests that the nett Doppler shift decreases as the aircraft path changes as α increases. That is the aircraft track changes from crossing the GCL at right angles toward flying down the GCL nett Doppler reduces. The time of acquisition of the signal increases as the aircraft spends more time in the cone of illumination of the sender during which it is also "visible" to the receiver.

To confirm this proposition, a series of computations were made at different crossing angles and a summary of the results are given in graphical form in Figure 4. The graph shows the expected cosine shape with maximum nett Doppler occurring for flights crossing at right angles.

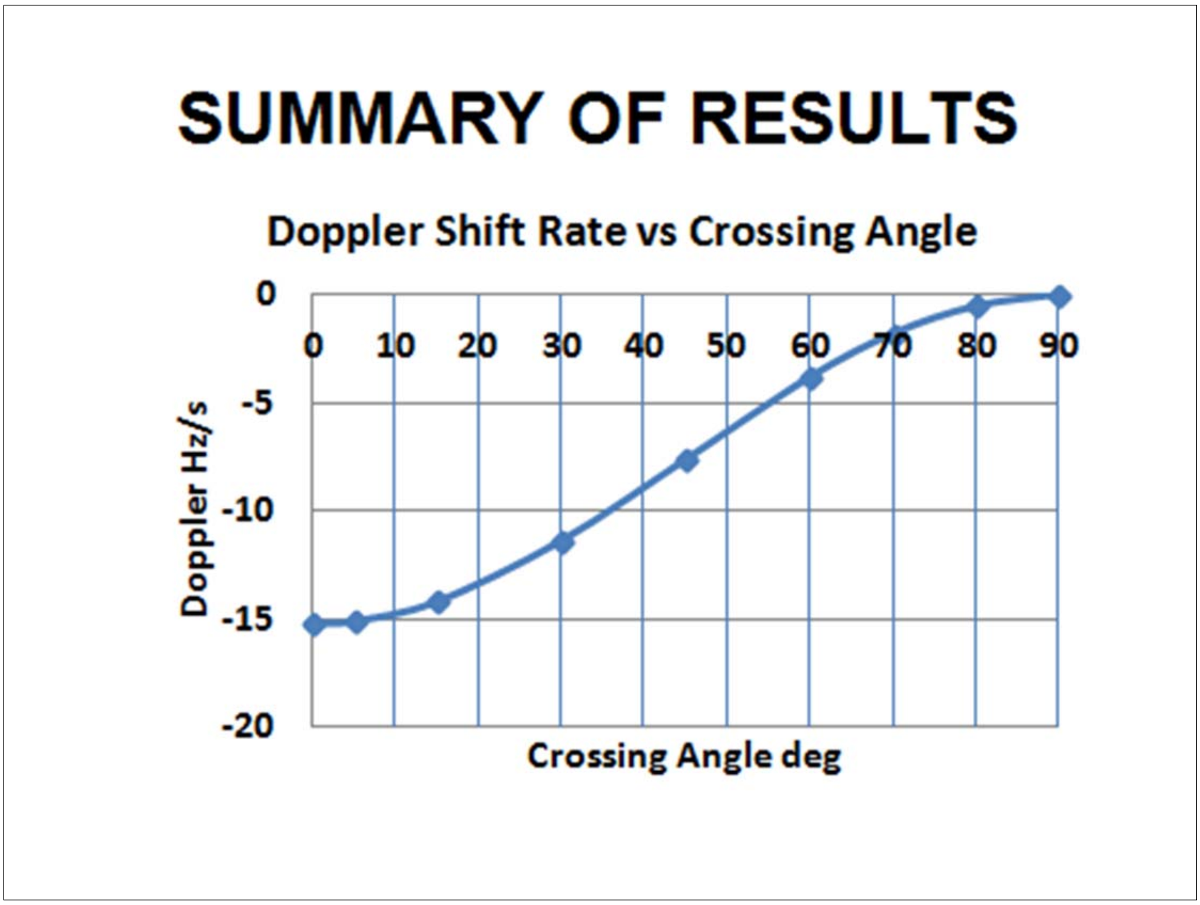

Figure 4. Variation of change in rate of Doppler shift against crossing angle of the GCL, α = 0° to α = 90°. For much of the useable flight path with small angles of inclination to the GCL (α near 90 °) the change in Doppler frequency is less than ± 2 Hz per second.

Most AE contacts involve a flight that crosses the GCL. Some contacts do occur with flights that fly parallel to the GCL or do not cross the GCL before the signal is lost. It is assumed that these will exhibit Doppler shifts that are less than occurs for flights that cross the GCL but more than for flights that are along of virtually along the GCL.

A separate geometry was adopted for the case of an aircraft flying down the GCL and is shown in outline in Figure 5.

Results are shown in Figure 6 with increased time for the AE contact being obvious as well as the minimal Doppler shift especially near mid path. When the aircraft is low on the horizon for one station it will appear higher at the other station and be much closer to it. This results in unequal angular changes and the linear relationship of Doppler frequency shift with distance along the flight Path becomes approximate rather than exact.

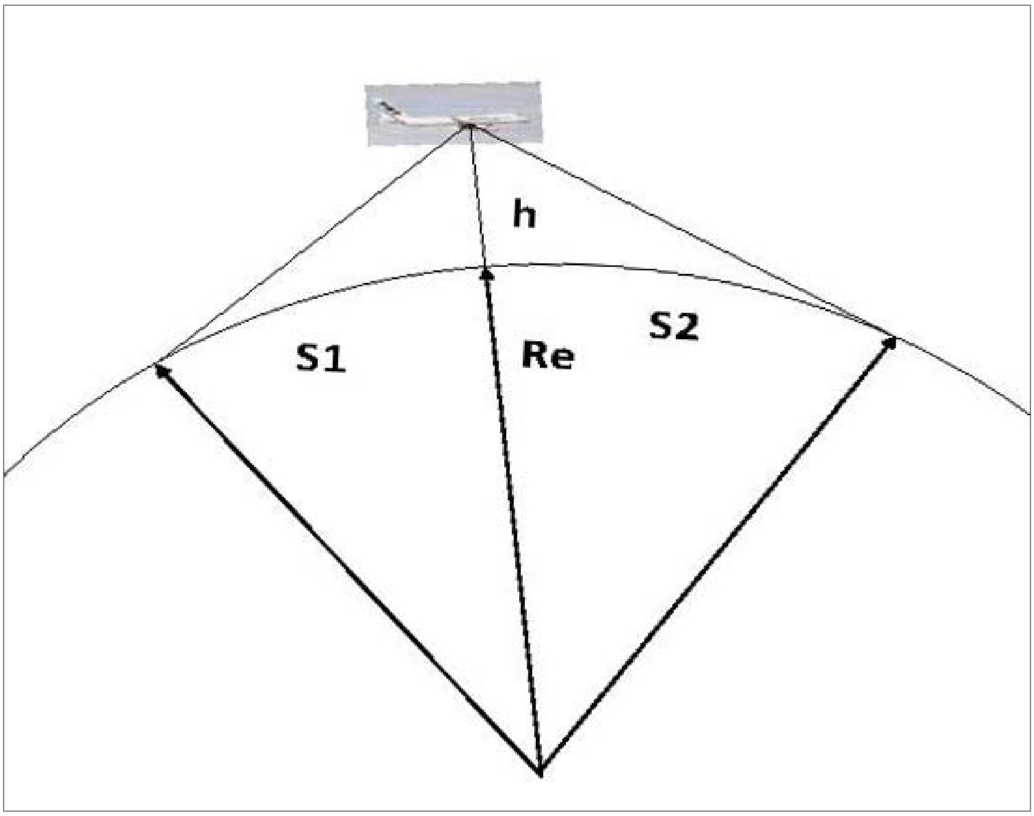

Figure 5. Basic geometry for GCL flight paths. S1 and S2 are the surface distances from station 1 and station 2 respectively to a point directly under the aircraft. S1 + S2 = 600 km. The aircraft height h is 36,000 ft, its speed is 500 knots and Re the effective radius of the earth is 8,495 km.

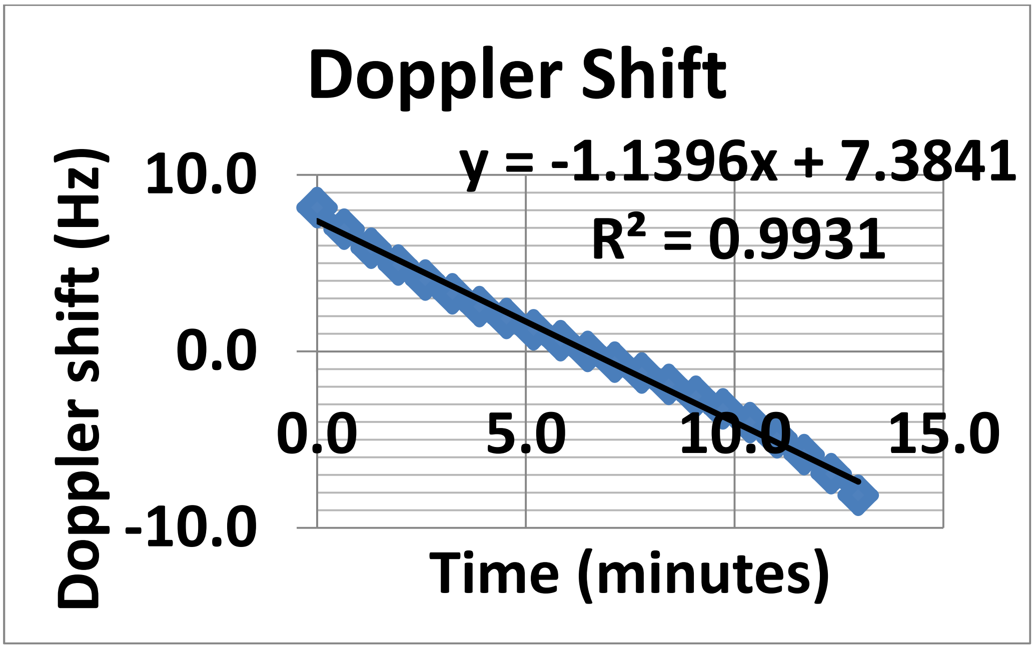

Figure 6. Doppler shift for a GCL flight path. The aircraft reaches the midpoint b between 6 and 7 minutes from the assumed acquisition point. The rate of change averages 1.14 Hz/minute.

For many paths such as those involving Sydney to Melbourne flights the aircraft is on ascent or descent during a significant part of the flight during which it is in mutual view of the two stations. Interesting Doppler shifts have been observed during the initial descent phase before signals are lost.

A series of calculations were done with some assumptions based on observations of aircraft position speed and altitude obtained by using the on-line program Flightradar24. These assumptions were that the flight line was on the GCL connecting the two stations, the descent angle was 4 degrees commencing at 36,000 feet and 190 km from the second station and there was no change in speed from 500 knots. The results are displayed in Figure 7. The descent speed is of course reduced at lower altitudes and the aircraft heading changes for approach to the airport but usually by this time the aircraft is no longer illuminated by rf from station 1. The assumptions are sufficiently representative for the few minutes of initial descent that still provide some AE.

Note the abrupt Doppler shift as soon as the aircraft starts to descend. The direction of travel of the aircraft changes quickly from level flight to a downward flight path. The southern station "sees" the descending aircraft approaching more directly than it was for the overhead flight path. That is the rate of approach is increased and consequently so is the Doppler shift. This quick rotation of the aircraft flight line causes a rapid change in the Doppler frequency. At the northern station the rate of change of distance does not change the same extent so the nett result is seen as a positive step change in Doppler which then decreases slowly as shown in Figure 7.

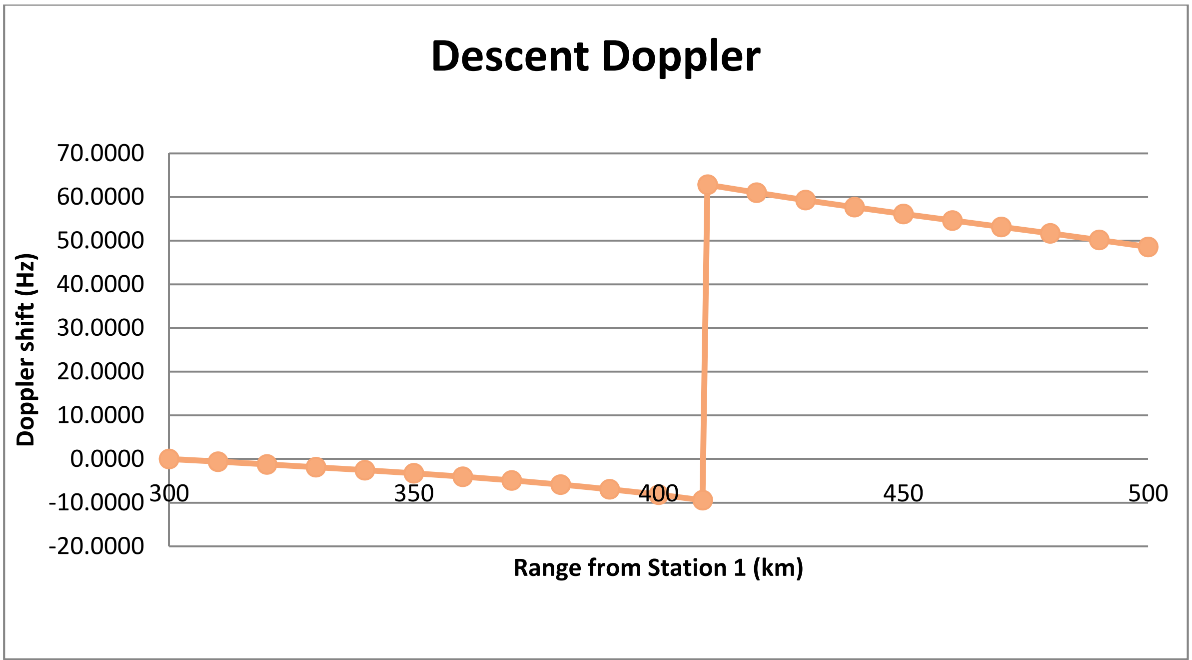

Figure 7. Doppler shift on descent for a typical Sydney Melbourne flight. The change is about 70 Hz. The descent is commenced at a range of 190 km, the descent angle is 4 degrees, the initial altitude is 36,000 feet and speed is maintained at 500 knot. The flight is assumed to be down the GCL connecting the two stations.

Observations

Some selected Spectran images of Doppler on a 10 GHz carrier are included to compare the calculated results to the observed results. Figure 8 shows two aircraft on the flight path crossing at an angle. The Doppler shift is about 40 Hz in 7 minutes. This is in good agreement with Figureures 4 and 6. We conclude that the flight path is near to but not exactly on the CGL.

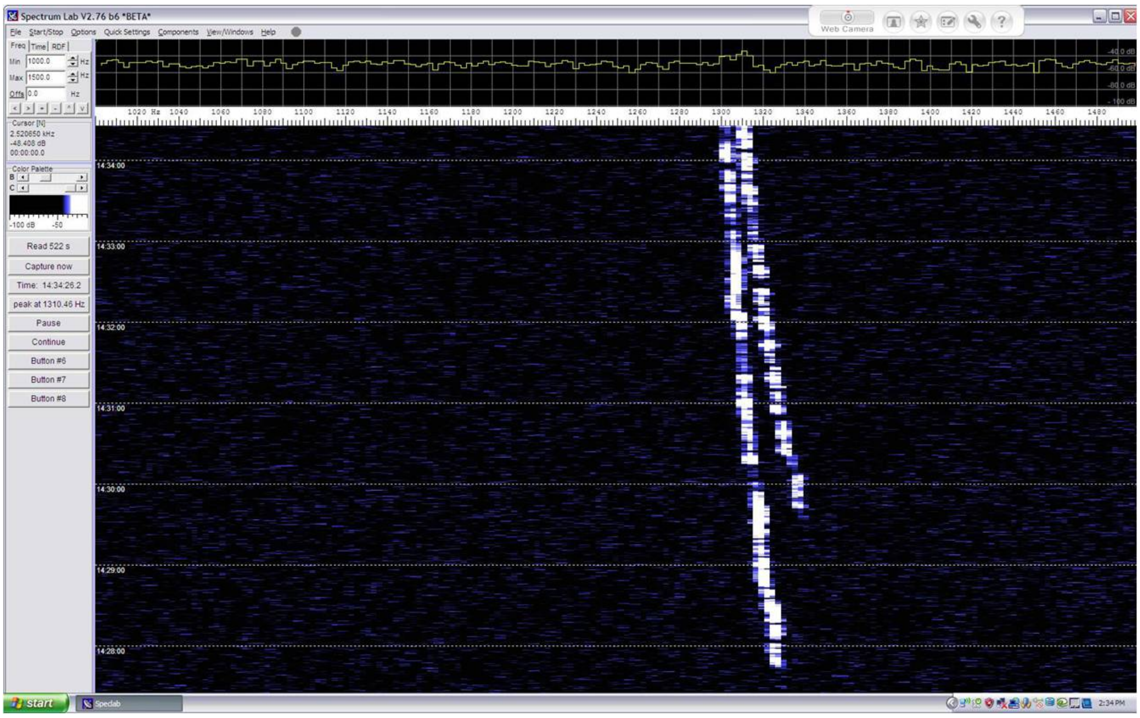

Figure 8. Observed Doppler Shift. Two aircraft are present one following the other and hence the Doppler shift also tracks with a time offset. The maximum shift seen here is about 40 Hz. Horizontal lines are 1 minute markers.

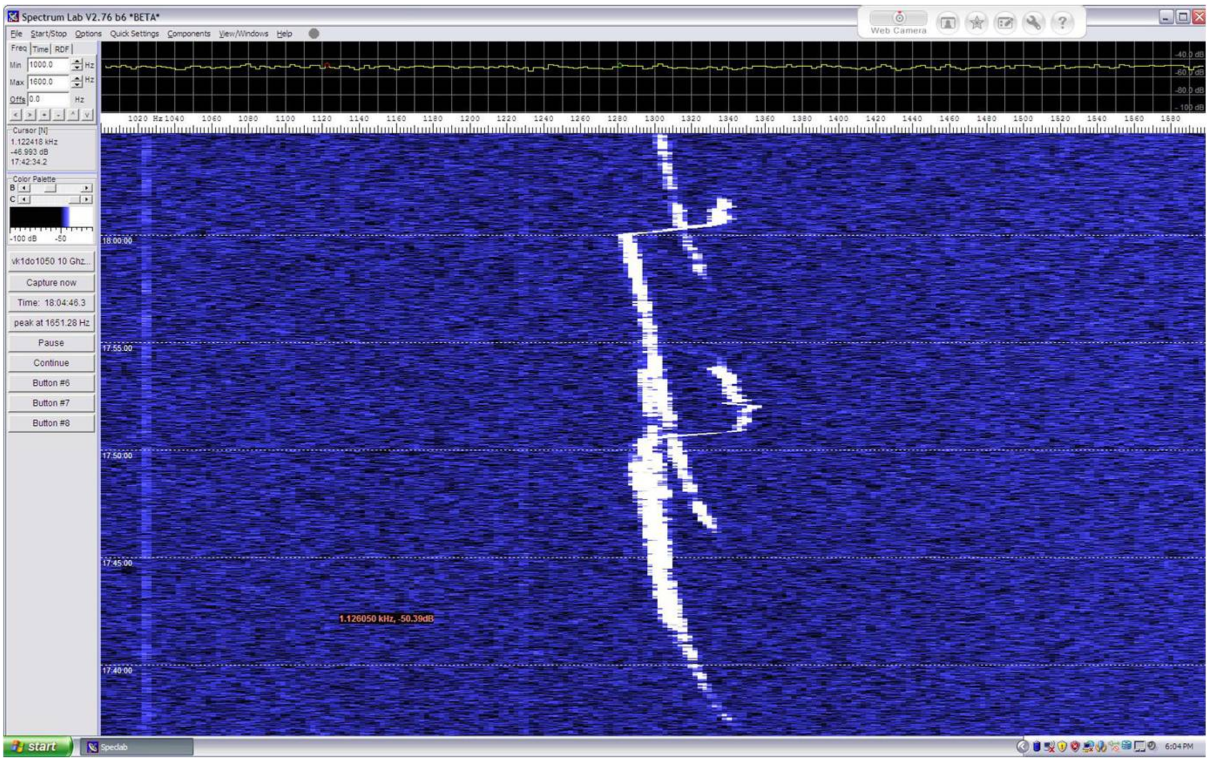

Figure 9. Observed Doppler Shift. Two aircraft are present one following the other and hence the Doppler shift also tracks with a time offset. They both descend giving a maximum shift of about 70 Hz. Horizontal lines are 1 minute markers.

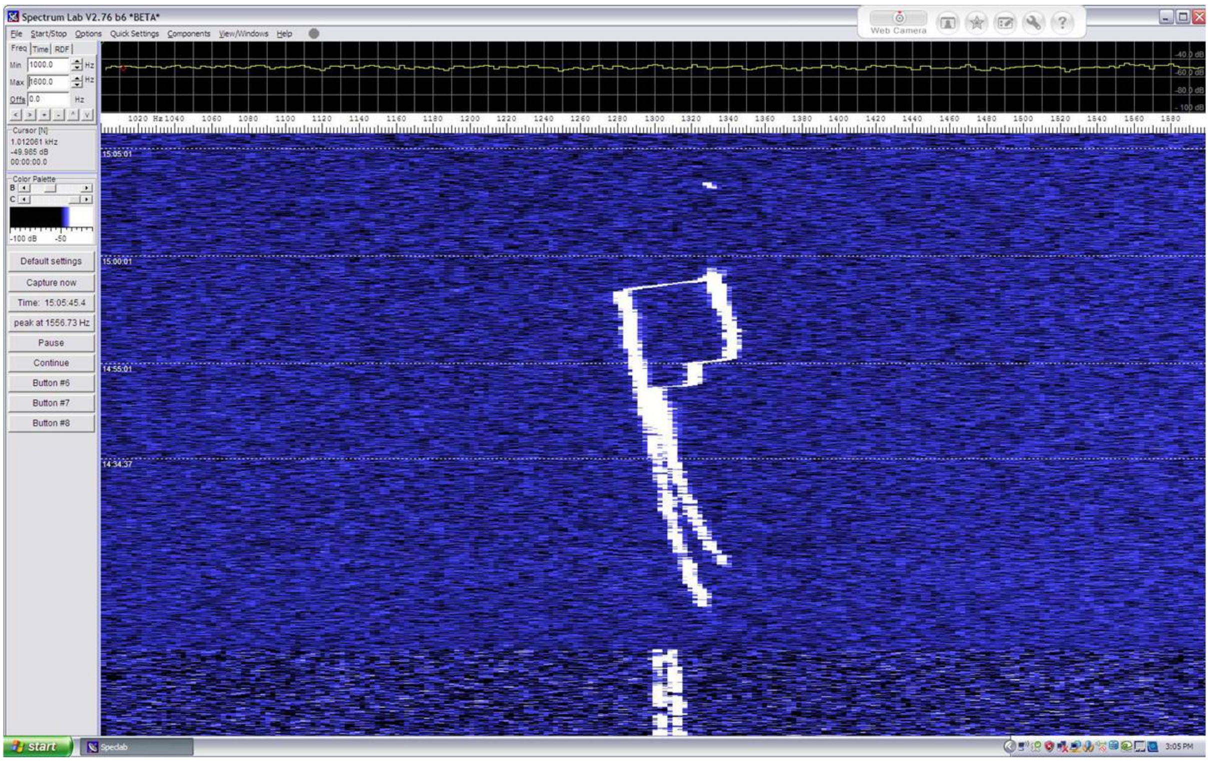

Figure 10. Observed Doppler Shift. Two aircraft are present one following the other and hence the Doppler shift also tracks with a time offset. They both descend giving a maximum shift of about 60 Hz. Horizontal lines are 1 minute markers. It appears that one aircraft changes to a steeper descent and the second does also but does it in two steps.

Figures 9 and 10 show aircraft entering the descent phase prior to loss of signal. The observed step changes in Doppler frequency and non-linear changes with time correlate well with the calculations.

Conclusions

Two dimensional geometry has been used to calculate the expected Doppler frequency shifts for 10 GHz transmissions using a typical AE path. It is possible to predict the approximate Doppler shift for any practical path using the spreadsheets developed. In practical situations where the antenna beamwidths are limited to around 3 degrees the Doppler shift will not exceed about 1 kHz for a path where the aircraft crosses the GCL at angles near 90 degrees for distances of 600 km or more between stations. As the flight path moves closer to an overhead flight along the GCL the Doppler shift reduces (and the acquisition time increases) while the aircraft maintains its altitude. Upon starting descent a step change in the Doppler shift occurs. This varies but typically is around 70 Hz.